

الدرس الثاني: الاشتقاق

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مثال 1-4:

$$1) f(x) = 4x + 1$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{(4(x+h) + 1) - (4x + 1)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{4x + 4h + 1 - 4x - 1}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{4h}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} 4$$

$$\Rightarrow \boxed{f'(x) = 4}$$

$$2) y = 1 - x$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{(1 - (x+h)) - (1 - x)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{1 - x - h - 1 + x}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{-h}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} -1$$

$$\Rightarrow \boxed{y' = -1}$$

$$3) f(x) = \frac{1}{2}x^2 - 1$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{\frac{1}{2}(x+h)^2 - 1 - (\frac{1}{2}x^2 - 1)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{\frac{1}{2}(x^2 + 2xh + h^2) - 1 - \frac{1}{2}x^2 + 1}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{\frac{1}{2}x^2 + xh + \frac{1}{2}h^2 - \frac{1}{2}x^2 + 1 - 1}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{xh + \frac{1}{2}h^2}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{x(\frac{1}{2}h)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} x + \frac{1}{2}(0)$$

$$\Rightarrow \lim_{h \rightarrow 0} x$$

$$\Rightarrow \boxed{f'(x) = x}$$

zoom

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$$4) y = \frac{2x+4}{6}$$

$$\Rightarrow y = \frac{2}{6}x + \frac{4}{6}$$

$$\Rightarrow y = \frac{1}{3}x + \frac{2}{3}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{\frac{1}{3}(x+h) + \frac{2}{3} - (\frac{1}{3}x + \frac{2}{3})}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{\cancel{\frac{1}{3}x} + \frac{1}{3}h + \cancel{\frac{2}{3}} - \cancel{\frac{1}{3}x} - \cancel{\frac{2}{3}}}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{\frac{1}{3}h}{h}$$

$$\Rightarrow y' = \frac{1}{3}$$

تأليف 5-7

$$5) f(x) = 4x^2, x=1$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{4(1+h)^2 - 4(1)^2}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{4(1^2 + 2h + h^2) - 4(1)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{4 + 8h + 4h^2 - 4}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{8h + 4h^2}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{h(8 + 4h)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} 8 + 4h$$

$$\Rightarrow \lim_{h \rightarrow 0} 8 + 4(0)$$

$$\Rightarrow \lim_{h \rightarrow 0} 8 + 0$$

$$\Rightarrow \lim_{h \rightarrow 0} 8$$

$$\Rightarrow f'(1) = 8$$

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6) $f(x) = 1 - x^2$, $x = -2$

$$\begin{aligned} &\Rightarrow \lim_{h \rightarrow 0} \frac{f(-2+h) - f(-2)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{1 - (-2+h)^2 - (1 - (-2)^2)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{1 - (4 - 4h + h^2) - 1 + (-2)^2}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{1 - 4 + 4h - h^2 - 1 + 4}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{4h - h^2}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{h(4-h)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} 4 - h \\ &\Rightarrow \lim_{h \rightarrow 0} 4 - (0) \\ &\Rightarrow \boxed{f'(-2) = 4} \end{aligned}$$

8) $f(x) = x^2 - 2x + 3$, $x = -1$

$$\begin{aligned} &\Rightarrow \lim_{h \rightarrow 0} \frac{f(-1+h) - f(-1)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{(-1+h)^2 - 2(-1+h) + 3 - ((-1)^2 - 2(-1) + 3)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{(-1^2 - 2h + h^2) + 2 - 2h + 3 - (1 + 2 + 3)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{1 - 2h + h^2 + 2 - 2h + 3 - 6}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{h^2 - 4h}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{h(h-4)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} (h-4) \\ &\Rightarrow \boxed{f'(-1) = -4} \end{aligned}$$

7) $f(x) = x^2 + x$, $x = 2$

$$\begin{aligned} &\Rightarrow \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{(2+h)^2 + (2+h) - (2^2 + 2)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{(4 + 4h + h^2) + (2+h) - (6)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{4 + 4h + h^2 + 2 + h - 6}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{h(5+h)}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} 5 + (0) \\ &\Rightarrow \boxed{f'(2) = 5} \end{aligned}$$

أختتم من ههنا

2) $y = x^{-11}$

$$\Rightarrow y' = -11x^{-12} = \boxed{\frac{-11}{x^{12}}}$$

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$$b) y = \frac{1}{x^5}$$

$$\Rightarrow y = x^{-5}$$

$$\Rightarrow y' = -5x^{-6} = \boxed{\frac{-5}{x^6}}$$

$$c) y = \sqrt[3]{x^5}$$

$$\Rightarrow y = x^{\frac{5}{3}}$$

$$\Rightarrow y' = \frac{5}{3} x^{\frac{2}{3}} = \boxed{\frac{5\sqrt[3]{x^2}}{3}}$$

اختتمت من فهمي من

$$2) y = \sqrt{x} + \frac{4}{x^2}$$

$$\Rightarrow y = x^{\frac{1}{2}} + 4x^{-2}$$

$$\Rightarrow y' = \frac{1}{2} x^{-\frac{1}{2}} - 8x^{-3}$$

$$\Rightarrow y' = \frac{1}{2x^{\frac{1}{2}}} - \frac{8}{x^3}$$

$$\Rightarrow \boxed{y' = \frac{1}{2\sqrt{x}} - \frac{8}{x^3}}$$

$$b) y = \frac{x^5 - 8x^6}{4x}$$

$$\Rightarrow y = \frac{x^5}{4x} - \frac{8x^6}{4x}$$

$$\Rightarrow y = \frac{1}{4}x^4 - 2x^5$$

$$\Rightarrow \boxed{y' = x^3 - 10x^4}$$

تأليف 9-14

$$9) y = 10x - \frac{6}{\sqrt{x}}$$

$$\Rightarrow y = 10x - 6x^{-\frac{1}{2}}$$

$$\Rightarrow y' = 10 + 12x^{-\frac{3}{2}}$$

$$\Rightarrow \boxed{y' = 10 + \frac{12}{x^{\frac{3}{2}}}}$$

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$$10) y = x^8 - x^{-8}$$

$$\Rightarrow y' = 8x^7 + 8x^{-9}$$

$$\Rightarrow y' = 8x^7 + \frac{8}{x^9}$$

$$11) y = 9x^{-2} + 3\sqrt{x}$$

$$\Rightarrow y = 9x^{-2} + 3x^{\frac{1}{2}}$$

$$\Rightarrow y' = -18x^{-3} + \frac{3}{2}x^{-\frac{1}{2}}$$

$$\Rightarrow y' = \frac{-18}{x^3} + \frac{3}{2x^{\frac{1}{2}}}$$

$$\Rightarrow y' = \frac{-18}{x^3} + \frac{3}{2\sqrt{x}}$$

$$12) y = \frac{1+\sqrt{x}}{x}$$

$$\Rightarrow y = \frac{1}{x} + \frac{\sqrt{x}}{x}$$

$$\Rightarrow y = \frac{1}{x} + \frac{\sqrt{x}}{\sqrt{x}\sqrt{x}}$$

$$\Rightarrow y = x^{-1} + x^{-\frac{1}{2}}$$

$$\Rightarrow y' = -x^{-2} - \frac{1}{2}x^{-\frac{3}{2}}$$

$$\Rightarrow y' = -\frac{1}{x^2} - \frac{1}{2x^{\frac{3}{2}}}$$

$$\Rightarrow y' = -\frac{1}{x^2} - \frac{1}{2\sqrt{x^3}}$$

$$13) y = \frac{6}{x^3} + \frac{2}{x^2} - 3$$

$$\Rightarrow y = 6x^{-3} + 2x^{-2} - 3$$

$$\Rightarrow y' = -18x^{-4} - 4x^{-3} - 0$$

$$\Rightarrow y' = \frac{-18}{x^4} - \frac{4}{x^3}$$

الدرس الثاني: الاشتقاق

$$14) y = 20x^5 + 3\sqrt[3]{x} + 17$$

$$\Rightarrow y = 20x^5 + 3x^{\frac{1}{3}} + 17$$

$$\Rightarrow y' = 100x^4 + x^{-\frac{2}{3}} + 0$$

$$\Rightarrow y' = 100x^4 + \frac{1}{x^{\frac{2}{3}}}$$

$$\Rightarrow y' = 100x^4 + \frac{1}{\sqrt[3]{x^2}}$$

أتحقق من فهمي 72

$$y = 8x - \frac{1}{x}$$

$$\Rightarrow y = 8x - x^{-1}$$

$$\Rightarrow y' = 8 + x^{-2}$$

$$\Rightarrow y' = 8 + \frac{1}{x^2}$$

$$\Rightarrow y - y_1 = m(x - x_1)$$

$$\Rightarrow y + 2 = m(x - \frac{1}{4})$$

$$\Rightarrow m = y' = 8 + \frac{1}{(0.25)^2} = 24$$

$$\Rightarrow y + 2 = 24(x - \frac{1}{4})$$

$$\Rightarrow y = 24x - 6 - 2$$

$$\Rightarrow y = 24x - 8 \quad \text{معادلة المماس}$$

$$\Rightarrow y + 2 = \frac{-1}{24}(x - \frac{1}{4})$$

$$\Rightarrow y = \frac{-x}{24} + \frac{1}{96} - 2$$

$$\Rightarrow y = \frac{-x}{24} - \frac{191}{96} \quad \text{معادلة التماس على المحاور}$$

تأليف 15-16

الدرس الثاني: الاشتقاق

$$15) y = (4)^2 - (4) = 12$$

$$\Rightarrow y = x^2 - x$$

$$\Rightarrow y' = 2x - 1$$

$$\Rightarrow y' = m = 2(4) - 1$$

$$\Rightarrow y' = m = 7$$

$$\Rightarrow y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 12 = 7(x - 4)$$

$$\Rightarrow y = 7x - 28 + 12$$

$$\Rightarrow y = 7x - 16$$
 معادلة المماس

$$16) m = -\frac{1}{7}$$

$$\Rightarrow y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 12 = -\frac{1}{7}(x - 4)$$

$$\Rightarrow y = \frac{-x}{7} + \frac{4}{7} + 12$$

$$\Rightarrow y = \frac{-x + 88}{7}$$

معادلة العمودي
على المماس

أتحقق من أنني على حق

$$2) f(x) = 1 - \sqrt{x}$$

$$\Rightarrow f(x) = 1 - x^{\frac{1}{2}}$$

$$\Rightarrow f'(x) = 0 - \frac{1}{2}x^{-\frac{1}{2}}$$

$$\Rightarrow f'(x) = \frac{-1}{2x^{\frac{1}{2}}}$$

$$\Rightarrow f'(x) = \frac{-1}{2\sqrt{x}}$$

$$\Rightarrow \frac{-1}{2\sqrt{x}} = \frac{-1}{2\sqrt{x}} \times \frac{2\sqrt{x}}{2\sqrt{x}}$$

zoom

الدرس الثاني: الاشتقاق

$$\Rightarrow -\frac{1}{2}\sqrt{x} = -1 \times 2$$

$$\Rightarrow \sqrt{x} = 2^2$$

$$\Rightarrow \boxed{x = 4}$$

$$\Rightarrow f(4) = 1 - \sqrt{4}$$

$$\Rightarrow f(4) = 1 - (2)$$

$$\Rightarrow \boxed{f(4) = -1}$$

$$\Rightarrow \boxed{(4, -1)}$$

تأليف : 23-24

$$23) P = (1, 5)$$

$$\Rightarrow y = 6x - x^2$$

$$\Rightarrow y' = 6 - 2x$$

$$\Rightarrow m = y' = 6 - 2(1) = \boxed{4}$$

$$\Rightarrow y - 5 = 4(x - 1)$$

$$\Rightarrow y = 4x - 4 + 5$$

$$\Rightarrow \boxed{y = 4x + 1}$$
 معادلة الخط

$$24) m = \boxed{\frac{-1}{4}}$$

$$\Rightarrow y - 5 = \frac{-1}{4}(x - 1)$$

$$\Rightarrow y = \frac{-x}{4} + \frac{1}{4} + 5$$

$$\Rightarrow \boxed{y = \frac{-x + 21}{4}}$$
 معادلة العردي على المماس

الدرس الثاني: الاشتقاق

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b) $f(x) = -x^3 + 3x^2 - 2$

$\Rightarrow f'(x) = -3x^2 + 6x \Rightarrow f'(x) = m = 0$

$\Rightarrow -3x^2 + 6x = 0$

$\Rightarrow x(-3x + 6) = 0$ ويمكن حساب $(-3x)$ كما هو موضح

$\Rightarrow -3x + 6 = 0 \Rightarrow x = 0$

$\Rightarrow \frac{-3x}{-3} = \frac{-6}{-3} \Rightarrow x = 2$

$\Rightarrow f(0) = -(0)^3 + 3(0)^2 - 2 \Rightarrow f(0) = 0 + 0 - 2$

$\Rightarrow f(0) = -2$

$\Rightarrow f(2) = -(2)^3 + 3(2)^2 - 2 \Rightarrow f(2) = -8 + 12 - 2$

$\Rightarrow f(2) = 2$

$\Rightarrow (0, -2)$

$\Rightarrow (2, 2)$

تأليف 20-22

20) $f(x) = x^2 - x - 12$

$\Rightarrow f'(x) = 2x - 1$

$\Rightarrow m = 3$

$\Rightarrow f'(x) = m$

$\Rightarrow 2x - 1 = 3$

$\Rightarrow 2x = \frac{4}{2}$

$\Rightarrow x = 2$

$\Rightarrow f(2) = (2^2) - (2) - 12 = 4 - 14 = -10$

$\Rightarrow (2, -10)$

$\Rightarrow y - y_1 = m(x - x_1)$

$\Rightarrow y - (-10) = 3(x - 2)$

$\Rightarrow y + 10 = 3(x - 2)$

$\Rightarrow y = 3(x - 2) - 10$

$\Rightarrow y = 3x - 6 - 10$

$\Rightarrow y = 3x - 16$ معادلة المماس

Zoom

الدرس الثاني: الاشتقاق

$$21) f(x) = x^3 - 4x^2 - 4$$

$$\Rightarrow f'(x) = 3x^2 - 8x \rightarrow f'(x) = m = 0$$

$$\Rightarrow 3x^2 - 8x = 0$$

$$\Rightarrow x(3x - 8) = 0$$

$$\Rightarrow x = 0$$

$$\Rightarrow 3x - 8 = 0$$

$$\Rightarrow f(0) = (0)^3 - 4(0)^2 - 4$$

$$\Rightarrow 3x = 8$$

$$\Rightarrow f(0) = -4$$

$$\Rightarrow x = \frac{8}{3}$$

$$\Rightarrow (0, -4)$$

$$\Rightarrow f\left(\frac{8}{3}\right) = 3\left(\frac{8}{3}\right)^3 - 4\left(\frac{8}{3}\right)^2 - 4$$

$$\Rightarrow f\left(\frac{8}{3}\right) = \frac{220}{9}$$

$$\Rightarrow \left(\frac{8}{3}, \frac{220}{9}\right)$$

$$22) f(x) = 5x^2 - 49x + 12$$

$$\Rightarrow f'(x) = 10x - 49$$

$$\Rightarrow m = 1$$

$$\Rightarrow f'(x) = m$$

$$\Rightarrow 10x - 49 = 1$$

$$\Rightarrow \frac{10x}{10} = \frac{50}{10}$$

$$\Rightarrow x = 5$$

$$\Rightarrow f(5) = 5(5)^2 - 49(5) + 12$$

$$\Rightarrow f(5) = 5(25) - 245 + 12$$

$$\Rightarrow f(5) = 125 - 233$$

$$\Rightarrow f(5) = -108$$

$$\Rightarrow (5, -108)$$